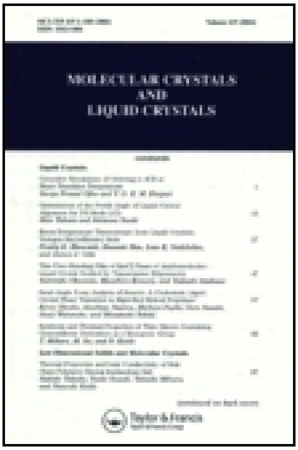

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Propagating Plane Disinclination Surfaces in Nematic Liquid Crystals

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Abstract—A theoretical study is made of the propagation through nematic liquid crystals of surfaces carrying a discontinuity in the orientation. The continuum theory for liquid crystals is used. It is found that such surfaces are able to propagate without suffering damping, in contrast with other types of wave in liquid crystals.

1. Introduction

Theoretical studies of the propagation of various different types of waves in liquid crystals have been published: weak waves carrying discontinuities in the second spatial derivatives of the orientation by Ericksen[1]; twist waves by Ericksen[2]; infinitesimal plane progressive sinusoidal waves by the Orsay Group,[3] Martinoty and Candau[4] and Currie[5]; and plane progressive surface waves by Papoular and Rapini.[6,7] All these waves are damped, this being shown for weak waves by Currie,[8] and the damping would seem to be fairly heavy. The purpose of this paper is to examine a type of wave that may be able to propagate without damping.

The wave is best described as a propagating disinclination surface. It is a surface across which the orientation in the liquid crystal suffers a discontinuity. The velocity, however, is continuous, only the acceleration having a discontinuity; so the wave has none of the characteristics of a shock wave.

The continuum theory for nematic liquid crystals proposed by Leslie [9,10,11] is adopted. It is found that disinclination surfaces are able to propagate without damping, but that some of their properties and their wave speeds depend on the assumptions made about the vector $\boldsymbol{\beta}$. This vector arises in the theory from the assumption that the magnitude of the director describing the orientation in the material is fixed. Since $\boldsymbol{\beta}$ does not occur in the local field equations it can be determined only through boundary conditions. Thus it is not clear how $\boldsymbol{\beta}$ should be restricted at the wave. We explore all possibilities and show that even if $\boldsymbol{\beta}$ is zero throughout the material there is still one direction along which an undamped disinclination surface can propagate.

It is possible that additional effects not considered here may modify the behaviour of these waves. For example, it may be necessary to attribute a surface energy to the disinclination. In the absence of such further effects, the waves discussed here are exact solutions of the governing equations. As such they may be of some experimental interest.

2. Governing Equations

The equations governing the motion of an incompressible nematic liquid crystal are taken to be those proposed by Leslie.[9, 10, 11] In this theory the orientation in the material is specified by a unit vector $\mathbf{d}$ called the director. In Cartesian tensor notation the restriction on the magnitude $\mathbf{d}$ and the incompressibility condition on the velocity vector $\mathbf{v}$ give

$$d_i d_i = 1,\ d_i d_{i,k} = 0,\ d_i \dot{d}_i = 0,\ v_{i,i} = 0, \tag{2.1}$$

where the superposed dot denotes material derivative. The balance laws for a material volume V bounded by a surface A are

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V (\tfrac{1}{2}\rho v_i v_i + W + TS + \tfrac{1}{2}\rho_1 \dot{d}_i \dot{d}_i)\,\mathrm{d}V = \int_V (r + F_i v_i + G_i \dot{d}_i)\,\mathrm{d}V + \int_A (v_i \sigma_{ij} + \dot{d}_i \pi_{ij} - q_j)\,\nu_j\,\mathrm{d}A,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho v_i\,\mathrm{d}V = \int_V F_i\,\mathrm{d}V + \int_A \sigma_{ij}\nu_j\,\mathrm{d}A,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_1 \dot{d}_i\,\mathrm{d}V = \int_V (g_i + G_i)\,\mathrm{d}V + \int_A \pi_{ij}\nu_j\,\mathrm{d}A. \tag{2.2}$$

Here ρ is the constant density, ρ_1 a positive inertial constant. $\mathrm{d}/\mathrm{d}t$ is the material derivative. W is the free energy, T the temperature and r the heat supply per unit volume. ν is the unit normal to the surface A. **F** and **G** are the external body force and external director body force. The intrinsic body force **g**, the stress tensor $\boldsymbol{\sigma}$, the director stress $\boldsymbol{\pi}$, the entropy S and the heat flux vector **q** are given by

$$\begin{aligned}
\pi_{ij} &= \beta_j d_i + \frac{\partial W}{\partial d_{i,j}}, \quad S = \frac{\partial W}{\partial T}, \\
q_i &= \kappa_1 T_{,i} + \kappa_2 d_j T_{,j} d_i, \\
g_i &= \gamma d_i - (\beta_j d_i)_{,j} - \frac{\partial W}{\partial d_i} + \lambda_1 N_i + \lambda_2 d_j A_{ji}, \\
\sigma_{ij} &= -p\delta_{ij} - \frac{\partial W}{\partial d_{k,j}} d_{k,i} + \mu_1 d_k d_p A_{kp} d_i d_j + \mu_2 d_j N_i \\
&\quad + \mu_3 d_i N_j + \mu_4 A_{ij} + \mu_5 d_j d_k A_{ki} + \mu_6 d_i d_k A_{kj}. \qquad (2.3)
\end{aligned}$$

p is an undetermined pressure, γ an undetermined scalar and $\boldsymbol{\beta}$ an undetermined vector. Both γ and $\boldsymbol{\beta}$ arise from the assumption of constant director magnitude. The viscosities $\mu_1 \ldots \mu_6$ and the conductivities κ_1, κ_2 are functions of temperature alone. $\lambda_1 = \mu_2 - \mu_3$ and $\lambda_2 = \mu_5 - \mu_6$. **A** and **N** are defined by

$$2A_{ij} = v_{i,j} + v_{j,i}, \; N_i = \dot{d}_i + \tfrac{1}{2}(v_{j,i} - v_{i,j}) d_j. \qquad (2.4)$$

We shall assume that the free energy W has the form proposed by Frank.[12]

$$\begin{aligned}
2W &= (\alpha_1 - \alpha_2 - \alpha_4) d_{i,i} d_{j,j} + \alpha_2 d_{i,j} d_{i,j} + \alpha_4 d_{i,j} d_{j,i} \\
&\quad + (\alpha_3 - \alpha_2) d_i d_j d_{k,i} d_{k,j} \qquad (2.5)
\end{aligned}$$

$\alpha_1 \ldots \alpha_4$ are functions of temperature alone and are assumed to satisfy the restrictions imposed by Ericksen[13]

$$\alpha_1 \geqslant 0, \; \alpha_2 \geqslant 0, \; \alpha_3 \geqslant 0, \; |\alpha_4| \leqslant \alpha_2, \; \alpha_2 + \alpha_4 < \alpha_1. \qquad (2.6)$$

In addition to the balance laws (2.2) we adopt the usual entropy inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V S \,\mathrm{d}V - \int_V \frac{r}{T} \mathrm{d}V + \int_A \frac{q_i \nu_i}{T} \mathrm{d}A \geqslant 0. \qquad (2.7)$$

In regions in which all functions are sufficiently smooth, Eqs. (2.2) and (2.7) give, after some manipulation, the local equations

$$\begin{aligned}
\dot{W} + T\dot{S} + \dot{T}S &= r - q_{i,i} + \sigma_{ij} v_{i,j} + \pi_{ij} \dot{d}_{i,j} - g_i \dot{d}_i, \\
\rho \dot{v}_i &= F_i + \sigma_{ij,j}, \\
\rho_1 \ddot{d}_i &= g_i + G_i + \pi_{ij,j}, \\
T(T\dot{S} - r + q_{i,i}) - T_{,i} q_i &\geqslant 0. \qquad (2.8)
\end{aligned}$$

3. Jump Conditions

Consider a singular surface across which **v** and T are continuous but **d** and the derivatives of **d**, **v** and T are discontinuous. We assume that the surface is plane and is propagating into a region in which **v** is zero and **d** and T are constant. U will denote the speed of propagation of the surface and **n** the unit normal to the surface. Square brackets will be used to denote the jump in a quantity across the surface, the jump being the value ahead of the discontinuity minus the value behind.

The balance equations (2.2) and the entropy inequality (2.7) lead to the following conditions across the discontinuity (Truesdell and Toupin,[14] Secs. 193, 258):

$$[W+TS+\tfrac{1}{2}\rho_1\dot{d}_i\dot{d}_i]\,U+[v_i\sigma_{ij}+\dot{d}_i\pi_{ij}-q_j]\,n_j=0\,,$$
$$[\sigma_{ij}]n_j=0,\ [\rho_1\dot{d}_i]U+[\pi_{ij}]n_j=0\,,$$
$$[S]TU-[q_j]n_j\leqslant 0\,. \tag{3.1}$$

We define quantities **A**, B, **C** and **D** by

$$[\mathrm{v}_{i,k}]n_k=A_i,\ [T_{,i}]n_i=B,\ [d_{i,k}]n_k=C_i,\ [d_i]=D_i\,. \tag{3.2}$$

The compatibility conditions then give (Truesdell and Toupin,[14] Secs. 175–181), assuming that **D** is constant along the surface,

$$[v_{i,k}]=A_in_k,\ [\dot{v}_i]=-UA_i,\ [T_{,i}]=Bn_i\,,$$
$$[d_{i,k}]=C_in_k,\ [\dot{d}_i]=E_i-UC_i\,, \tag{3.3}$$

where $\mathbf{E}=\delta\mathbf{D}/\delta t$ is the rate of change of **D** as seen by an observer moving with the surface.

From now on we shall use **d** exclusively to denote the orientation *behind* the discontinuity. Equations (2.1) and (3.3) give

$$D_i(D_i+2d_i)=0,\quad C_id_i=0\,,$$
$$E_id_i=0,\quad A_in_i=0\,, \tag{3.4}$$

The second and third of Eqs. (3.1), with the use of (2.3), give

$$\left\{2[p]-2\left[\frac{\partial W}{\partial d_{k,j}}\right]C_kn_j-A_kd_kd_jn_j(\mu_2+\mu_5)\right\}n_i$$
$$=d_i\{2\mu_3(E_p-UC_p)n_p+A_pd_p(2\mu_1d_kn_kd_jn_j+\mu_3+\mu_6)\}$$
$$+2(E_i-UC_i)\mu_2d_pn_p+A_i\{\mu_4+(\mu_5+\mu_2)(d_kn_k)^2\}\,, \tag{3.5}$$

and

$$\rho_1 U(E_i - UC_i) + C_i\{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2\} + n_i(\alpha_1 - \alpha_2)C_j n_j = -[\beta_j n_j]d_i - \beta_0 D_i, \qquad (3.6)$$

where $\beta_0 = \beta_j n_j$ evaluated ahead of the discontinuity in the region of constant **d**. Also, the first equation of (3.1), together with (2.3) and (3.6), yields

$$\begin{aligned}2\{\kappa_1 + \kappa_2(d_j n_j)^2\}B &= UT(\alpha_1' - \alpha_2')(C_j n_j)^2 + UT\{\alpha_2' + (\alpha_3' - \alpha_2')(d_j n_j)^2\}C_i C_i \\ &\quad - \{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2\}E_i C_i - (\alpha_1 - \alpha_2)E_i n_i C_j n_j \\ &\quad + \beta_0(E_i - UC_i)D_i, \qquad (3.7)\end{aligned}$$

where $\alpha_i' = \mathrm{d}\alpha_i/\mathrm{d}T$. Finally, the inequality in (3.1), with the use of (2.3), (3.3) and (3.7), becomes

$$\{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2\}E_i C_i + (\alpha_1 - \alpha_2)E_i n_i C_j n_j - \beta_0(E_j - UC_i)D_i \geqslant 0. \qquad (3.8)$$

We shall now concentrate on the case when **A**, B, **C** and **D** are constant and do not vary as the wave propagates through the material. If such a wave is possible then it is likely to be of experimental interest since it will persist in time and not decay. Since **D** is constant and the wave is plane, $\mathbf{E} = 0$. Elimination of $[p]$ from (3.5) now yields

$$\begin{aligned}&(d_i - n_i d_s n_s)\{A_p d_p(2\mu_1 d_k n_k d_j n_j + \mu_3 + \mu_6) - 2\mu_3 UC_p n_p\} \\ &\quad + A_i\{\mu_4 + (\mu_5 - \mu_2)(d_j n_j)^2\} + (n_i C_j n_j - C_i)2\mu_2 U d_p n_p = 0. \qquad (3.9)\end{aligned}$$

Equation (3.6), with $\mathbf{E} = 0$, gives

$$C_i\{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2 - \rho_1 U^2\} + n_i(\alpha_1 - \alpha_2)C_j n_j = -[\beta_j n_j]d_i - \beta_0 D_i, \qquad (3.10)$$

and (3.7) becomes

$$\begin{aligned}2\{\kappa_1 + \kappa_2(d_j n_j)^2\}B &= UT(\alpha_1' - \alpha_2')(C_j n_j)^2 \\ &\quad + UT\{\alpha_2' + (\alpha_3' - \alpha_2')(d_j n_j)^2\}C_i C_i - \beta_0 UC_i D_i. \qquad (3.11)\end{aligned}$$

With $\mathbf{E} = 0$, the inequality (3.8) becomes

$$\beta_0 UC_i D_i \leqslant 0. \qquad (3.12)$$

In addition to the above conditions derived from the integral balance laws (2.2) there are also conditions coming from the local equations (2.8). Since $\mathbf{C}$ is constant there exists $\mathbf{M}$ such that (Truesdell and Toupin,[14] Secs. 175–181)

$$[d_{i,jk}] = M_i n_j n_k,\ [\dot{d}_{i,j}] = -UM_i n_j,\ [\ddot{d}_i] = U^2 M_i,$$
$$M_i d_i + C_i C_i = 0, \tag{3.13}$$

where (2.1) has been used. The first of Eqs. (2.8) determines $[T_{,ij}]n_i n_j$, the second (together with the incompressibility condition (2.1) on $\mathbf{v}$) determines $[p_{,j}]n_j$ and $[v_{i,jk}]n_j n_k$, while the inequality is satisfied identically if we assume that the material functions $\mu_1 \ldots \mu_6$, κ_1, κ_2 satisfy the inequalities proposed by Leslie.[15] With the use of (2.3), (3.3) and (3.13) the third equation gives an equation containing $\mathbf{M}$ and $[\gamma]$. Elimination of the arbitrary $[\gamma]$ yields

$$\begin{aligned}(\delta_{ij} - d_i d_j)(\gamma_0 D_j + [G_j]) &+ \tfrac{1}{2}(A_i - d_i A_j d_j)(\lambda_2 - \lambda_1) d_p n_p \\ &+ (n_i - d_i n_j d_j)\{\tfrac{1}{2}(\lambda_1 + \lambda_2) A_p d_p + (\alpha_1 - \alpha_2) M_p n_p \\ &- (\alpha_3 - \alpha_2) d_p n_p C_k C_k + B(\alpha_1' - \alpha_2') C_p n_p\} \\ &+ (M_i - d_i M_j d_j)\{\alpha_2 + (\alpha_3 - \alpha_2)(d_p n_p)^2 - \rho_1 U^2\} \\ &+ C_i\{2(\alpha_3 - \alpha_2) C_p n_p d_j n_j + \alpha_2' B + (\alpha_3' - \alpha_2')(n_p d_p)^2 B \\ &- \lambda_1 U\} = 0.\end{aligned} \tag{3.14}$$

Here γ_0 is the value of γ ahead of the wave.

4. The Propagation Condition

Consider now Eq. (3.10). This equation contains terms involving $\boldsymbol{\beta}$, the undetermined vector arising in the theory from the assumption of constant director magnitude. It can be seen from (2.8) that $\boldsymbol{\beta}$ does not enter into the local field equations. Thus $\boldsymbol{\beta}$ can be determined only through the boundary conditions, if any, on $\boldsymbol{\pi}$ or $\mathbf{g}$. It would appear reasonable to assume that $[\boldsymbol{\beta} \cdot \mathbf{n}]$ is arbitrary in (3.10). However, it is conceivable that the boundary conditions in both the region ahead of the wave and the region behind the wave require $\boldsymbol{\beta}$ constant across the wave. To allow for all possibilities we consider the following four cases:

(i) $[\boldsymbol{\beta} \cdot \mathbf{n}] = 0$, $\beta_0 = 0$,
(ii) $[\boldsymbol{\beta} \cdot \mathbf{n}]$ arbitrary, $\beta_0 = 0$,
(iii) $[\boldsymbol{\beta} \cdot \mathbf{n}] = 0$, $\beta_0 \neq 0$,
(iv) $[\boldsymbol{\beta} \cdot \mathbf{n}]$ arbitrary, $\beta_0 \neq 0$.

(i) $[\boldsymbol{\beta} \cdot \mathbf{n}] = 0$, $\beta_0 = 0$

In this case it follows at once from (3.10) that $\mathbf{C} = C\mathbf{n}$, assuming $\alpha_1 \neq \alpha_2$. But by (3.4) this is possible only if $\mathbf{d} \cdot \mathbf{n} = 0$. Thus, if $\boldsymbol{\beta} \cdot \mathbf{n}$ is zero on both sides of an undamped discontinuity the direction of propagation must be perpendicular to the orientation behind the wave. Of course, waves can propagate in other directions but it then follows from (3.6) that $\mathbf{E} \neq 0$ and so such waves will be damped.

For this undamped wave it is found from (3.9), (3.10) and (3.11) that

$$\rho_1 U^2 = \alpha_1, \qquad \mathbf{C} = C\mathbf{n}, \qquad \mathbf{A} = A\mathbf{d},$$
$$A(\mu_4 + \mu_3 + \mu_6) = 2\mu_3 UC, \qquad 2\kappa_1 B = T\alpha_1' C^2 \tag{4.1}$$

We note that the wave speed is real, by (2.6). The inequality (3.12) is satisfied identically.

If we now take the component of (3.14) parallel to $\mathbf{n}$ we find the equation

$$(\gamma_0 D_j + [G_j])n_j + \alpha_1' BC + \tfrac{1}{2}(\lambda_1 + \lambda_2)A - \lambda_1 UC = 0. \tag{4.2}$$

Substitution from (4.1) gives an equation determining C in terms of $\mathbf{D}$, T, $\mathbf{d}$ and $\mathbf{n}$.

$$\begin{aligned} &T(\alpha_1')^2(\mu_4 + \mu_3 + \mu_6)C^3 \\ &\quad + 2\kappa_1\{\lambda_2\mu_3 - \lambda_1(\mu_4 + \mu_6)\}UC \\ &\quad + 2\kappa_1(\mu_4 + \mu_3 + \mu_6)(\gamma_0 D_j + [G_j])n_j = 0. \end{aligned} \tag{4.3}$$

This equation is either a cubic in C or linear in C. Thus there is always at least one real root for C. If α_1 is independent of T then C is determined uniquely, but in general there may be three roots for C. However, all three waves will propagate with the same wave speed given by (4.1).

(ii) $[\boldsymbol{\beta} \cdot \mathbf{n}]$ arbitrary, $\beta_0 = 0$

Elimination of $[\boldsymbol{\beta} \cdot \mathbf{n}]$ from (3.10) gives

$$C_i\{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2 - \rho_1 U^2\} + (n_i - d_i n_p d_p)(\alpha_1 - \alpha_2)C_j n_j = 0. \tag{4.4}$$

This equation has two solutions for U and $\mathbf{C}$. With the corresponding value of $\mathbf{A}$ found from (3.9) the solutions are, for $\mathbf{n} \neq \mathbf{d}$,

$$\begin{aligned} &C_i = C(n_i - d_i n_j d_j), \qquad A_i = A(d_i - n_i d_j n_j), \\ &\rho_1 U^2 = \alpha_1 + (\alpha_3 - \alpha_1)(d_j n_j)^2, \\ &A\{\mu_4 + \mu_3 + \mu_6 + (2\mu_1 + \mu_5 - \mu_2 - \mu_3 - \mu_6)(d_j n_j)^2 - 2\mu_1(d_j n_j)^4\} \\ &\qquad = 2UC\{\mu_3 - (\mu_2 + \mu_3)(d_j n_j)^2\} \end{aligned} \tag{4.5}$$

and

$$C_i = C\epsilon_{ijk}n_j d_k, \qquad A_i = A\epsilon_{ijk}n_j d_k,$$
$$\rho_1 U^2 = \alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2,$$
$$A\{\mu_4 + (\mu_5 - \mu_2)(d_j n_j)^2\} = 2\mu_2 U C d_j n_j. \tag{4.6}$$

Thus for every direction of propagation $\mathbf{n}$ there are two types of wave. In the first type both $\mathbf{A}$ and $\mathbf{C}$ lie in the plane of $\mathbf{n}$ and $\mathbf{d}$, in the second type both $\mathbf{A}$ and $\mathbf{C}$ are perpendicular to this plane. The wave speeds are always real, by (2.6), and the inequality (3.12) is satisfied identically.

These solutions for the wave speed, and indeed for the discontinuities $\mathbf{A}$ and $\mathbf{C}$, are precisely those found by Ericksen[1] for weak waves. Thus it should be found experimentally that propagating disinclination surfaces behave much like weak waves, if $\boldsymbol{\beta}$ satisfies the assumed conditions. It is shown by Currie[5] that high-frequency sinusoidal waves also propagate with these same wave speeds.

Once again B is found from (3.11). As in case (i) above, Eq. (3.14) gives a cubic for C for each of the solutions (4.5) and (4.6). In the case of solution (4.5) the coefficient of C^2 is non-zero in this cubic. Thus, in the unlikely situation that the coefficient of C^3 vanishes, it is possible that there could be no real roots for C. There would then be no wave of the type under discussion. In general, however, there will always be at least one real root for C. For solution (4.6) the coefficient of C^2 in the cubic is zero, thus ensuring that there is always at least one real root for C.

(iii) $[\boldsymbol{\beta} \cdot \mathbf{n}] = 0, \beta_0 \neq 0$

Let $\mathbf{p} = \mathbf{d} \times \mathbf{n}\,|\mathbf{d} \times \mathbf{n}|^{-1}$. Then from (3.10) we find that

$$\rho_1 U^2 = \alpha_1 + (\alpha_3 - \alpha_2)(d_j n_j)^2 - \frac{(\alpha_1 - \alpha_2) d_j n_j D_i n_i}{D_k d_k}$$
$$C_i = \frac{\beta_0 D_j d_j}{\alpha_1 - \alpha_2}\left\{\frac{D_k p_k p_i}{D_q d_q - D_q n_q d_r n_r} - \frac{n_i - d_i n_k d_k}{d_q n_q (1 - d_r n_r d_s n_s)}\right\}, \tag{4.7}$$

where it has been assumed that neither $\mathbf{d} = \mathbf{n}$ nor $\mathbf{d} \cdot \mathbf{n} = 0$. $\mathbf{A}$ and B can be found from (3.9) and (3.11). Equation (3.14) now determines $\mathbf{M}$, and does not limit the magnitude of $\mathbf{C}$, in contrast with cases (i) and (ii).

From the inequality (3.12) and (4.7) we find the following condition for a positive wave speed U (so that the wave propagates into the region of constant orientation and zero velocity):

$$\frac{D_j d_j}{\alpha_1 - \alpha_2}\left\{\frac{(D_k p_k)^2}{D_q d_q - D_q n_q d_r n_r} - \frac{n_i D_i - d_i D_i n_k d_k}{d_q n_q(1 - d_r n_r d_s n_s)}\right\} \leqslant 0. \tag{4.8}$$

From (3.4) it follows that $\mathbf{d} \cdot \mathbf{D} \leqslant 0$. Moreover, $\mathbf{d}$ and $\mathbf{n}$ are unit vectors. Thus, if $\alpha_1 > \alpha_2$, a necessary condition following from (4.8) is

$$n_i D_i d_j n_j \leqslant 0. \tag{4.9}$$

A sufficient condition for (4.8) to hold, again with $\alpha_1 > \alpha_2$, is

$$D_i d_i - n_i D_i d_j n_j \geqslant 0. \tag{4.10}$$

The wave speed U in (4.7) is independent of β_0. Moreover, if $\alpha_3 > \alpha_1 > \alpha_2$ it follows from (2.6) that $\rho_1 U^2 > 0$, giving a real wave speed, provided that

$$D_i d_i\{1 + (d_j n_j)^2\} - n_i D_i d_j n_j \leqslant 0. \tag{4.11}$$

Thus sufficient conditions satisfying (4.8), and giving a real wave speed U when $\alpha_3 > \alpha_1 > \alpha_2$, are

$$D_i d_i\{1 + (d_j n_j)^2\} \leqslant n_i D_i d_j n_j \leqslant D_i d_i \leqslant 0. \tag{4.12}$$

Experiments by Saupe[16] show that for p-azoxyanisol $\alpha_3 > \alpha_1 > \alpha_2$.

Two particular limiting cases are of interest: if the orientation behind the discontinuity tends to a direction perpendicular to the direction of propagation, so that $\mathbf{d} \cdot \mathbf{n} \to 0$, then $\rho_1 U^2 \to \alpha_1$; if the orientation behind tends to the direction of propagation, so that $\mathbf{d} \to \mathbf{n}$, then $\rho_1 U^2 \to \alpha_3$.

(iv) $[\boldsymbol{\beta} \cdot \mathbf{n}]$ arbitrary, $\beta_0 \neq 0$

After the elimination of $[\boldsymbol{\beta} \cdot \mathbf{n}]$ from (3.10) we find

$$C_i\{\alpha_2 + (\alpha_3 - \alpha_2)(d_j n_j)^2 - \rho_1 U^2\} + (n_i - d_i n_p d_p)(\alpha_1 - \alpha_2) C_j n_j$$
$$= \beta_0 (d_i D_j d_j - D_i) \tag{4.13}$$

There are now insufficient conditions to determine $\mathbf{C}$ and U uniquely. U can take any value, other than the wave speeds given by (4.5) and (4.6). $\mathbf{C}$ is then determined by (4.13). This situation is similar to that encountered by Gurtin and Walsh[17] in their study of extrinsically induced weak waves in elastic bodies. Little more can be said in this case.

Finally, it should be noted that in all cases the wave speeds are independent of the body forces **F** and **G**, although the magnitude of the discontinuity **C** may depend on **G**, as in cases (i) and (ii).

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